we want to build $\tilde{H}(x-a)$

notice that $1 - H(x-a) = \tilde{H}(x-a)$

does the trick!

Even cooler than $H(x-a)$ is

$$\delta(x-a) = \frac{d}{dx} H(x-a).$$

$\delta(x-a)$ is called the "dirac-delta function"

back in the 1920's this thing made

mathematicians upset. Dirac was one of

the greatest Physicists ever.

$\delta(x-a)$ is an infinite spike at $x=a$

$$\delta(x-a) = \begin{cases} 0 & x \neq a \\ 1 & x = a \end{cases}$$

it can be defined by

$$\cancel{\delta(x-a) = \lim} \qquad \delta(x) = \lim_{\alpha \to 0} \frac{1}{a\sqrt{\pi}} e^{-\left(\frac{x}{a}\right)^2}$$

by golly thats a gaussian!

$\delta(x-a)$ is basically a gaussian distribution with zero variance!

we have a useful but seemingly trival formula

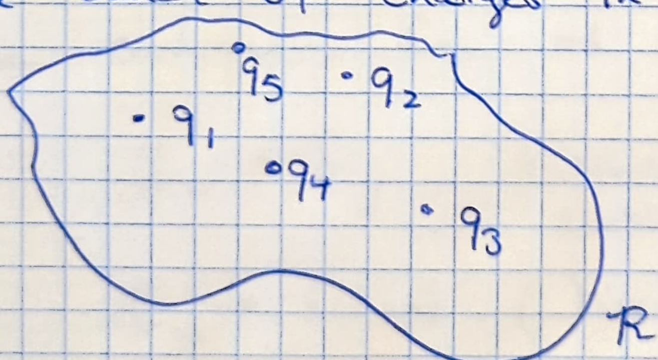
$$\int \delta(x-a) f(x)\, dx = f(a)$$

δ can pick out points of functions

why might this be useful?

suppose you want to use calculus on a bunch of discrete objects

e.g. a bunch of charges in some region R



then the charge density is

$$\rho(x) = \sum q_i\, \delta(x - a_i)$$

where a_i is the location of charge i

the total charge is just the integral of $\rho(x)$ over $\mathbb{R}$

$$Q = \int \rho(x)\, dV = \int \Sigma\, q_i\, \delta(x-a_i)\, dV$$

$$= q_1 + q_2 + \dots q_N$$

another useful way to use it is to kick a ball at $t = t_0$

$$F = |F|\, \delta(t - t_0)$$

these examples may seem trivial but it opens a box of a plethora of methods of computation
It's the continuous form of the Identity matrix

$$\mathbb{1}_N \rightarrow \delta_{ij} \quad (\text{Kronecker delta})$$

$N \times N$ matrix

$$\mathbb{1}_\infty \rightarrow \delta(x-a) \quad (\text{Dirac delta})$$