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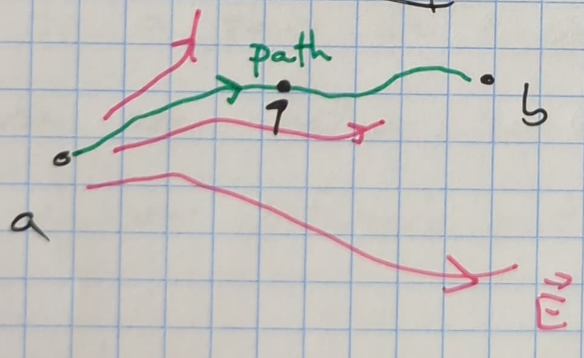
Electric Potential.

much like a mass held at a height h above the earth has a potential energy so too does a charge held in an electric field. if we release the charge then it will accelerate & convert this potential energy into Kinetic energy.

Additionally if we take a charge & move it a distance d in an electric field we will do <u>work</u> via $W = F \cdot \Delta x$

We can use the <u>work-energy</u> theorem to find the potential <u>energy</u> PE

$\boxed{W = -\Delta PE}$ along path a to b



where F is the force ON our charge q

$F = qE$

What is much more interesting is to consider the work done in moving <u>one</u> unit of charge (q = 1)

then $\boxed{\frac{\text{work}}{q} = -E\Delta x}$ where Δx is a small path.

this says we get energy out of a field which is kind of weird. This concept bothered 19th century physicists who had a newtonian view of the world.

Now, one might ask what the forces are that the moving charge exerts on anything else. For electrostatics we consider that the charge moves so slowly that the only force are coulombic, However, in electrodynamics really new and interesting things happen!

Now, we will define something analogous to gravitational potential & we call it electric potential our electric potential is ALWAYS defined with respect to a point we choose just like how we can chose we whether to use a table or the floor as our reference point near the earth's surface.

Define

$$\frac{\text{work}}{q} = \phi(P) = -E\Delta x = \frac{-\Delta PE}{q}$$

from point P_0 to P where P_0 is OUR chosen "ground" often we write this as

$$\bar{V} = \frac{PE}{q} = E \Delta x \quad \text{from } P_0 \text{ to } P$$

the electric potential is independent of the charge q. ~~Howeve~~ It is the potential energy a charge "would have" if placed at point P. However, depending on how $\bar{E}$ is produced there may be a charge q in the expression for the electric potential.

for example,

$$\bar{V}(r) = \frac{q}{4\pi\epsilon_0}\frac{1}{r}$$

this q is the one responsible for $\bar{E}$. If we now place a "test" charge q_t then the potential energy would be

$$q_t \frac{q}{4\pi\epsilon_0}\frac{1}{r} = PE = q_{test}\bar{V}$$

Now, the change in PE is $\Delta PE = -W$ and the "potential difference" is

$$\Delta \bar{V} = \bar{V}_P - \bar{V}_{P_0} = \frac{\Delta PE}{q}$$

This gives us a new unit to work with called the "Volt" (V)

$$1 \text{ Volt} = 1 \, J/C$$

we will also use the word "Voltage" to mean the potential difference b/w two points

for example,

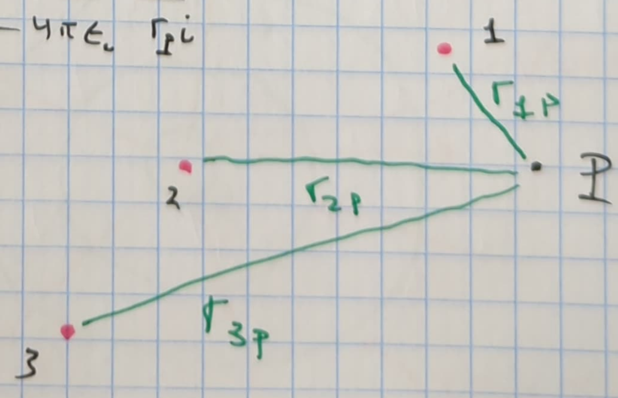
a 9 Volt battery has $\sim 9\,V$ between the two terminals.

Be mindful that Voltage is energy per charge. a 9 volt battery can store vastly different amounts of energy.

Finally, just as $\vec{E}$ from ~~au~~ many sources can be added up vectorially so too can potentials.

the potential at point P from many charges is

$$\phi(P) = \sum \frac{1}{4\pi\epsilon_0} \frac{1}{r_{Pi}}$$

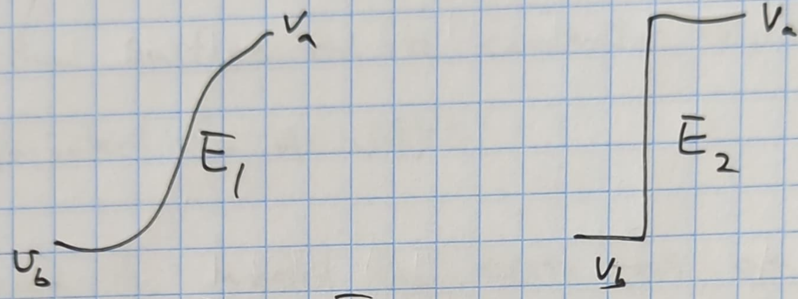


in general there is a beautiful and handy relation b/w

E & V

$$E = -\frac{\Delta V}{\Delta x}$$

E is sort of the "steepness" of a hill

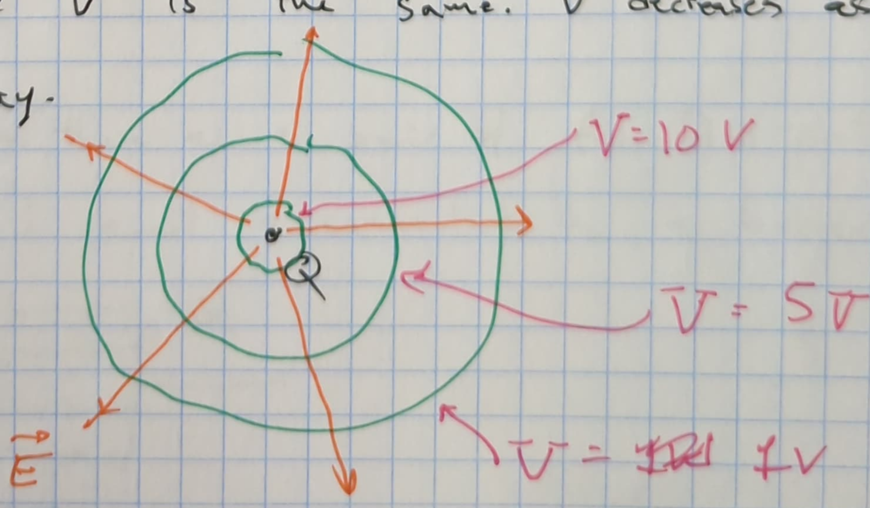


$E_2 > E_1$

E points in the direction of "decreasing" potential

Point charge Q | $V = \frac{kQ}{r}$

the potential can be thought of as concentric circles around Q called "equipotentials". they are circles where V is the same. V decreases as we move further away.



The equipotentials & field lines are $\perp$ at all points.

You can move a test charge around an equipotential with no work (modulo technical effects!)

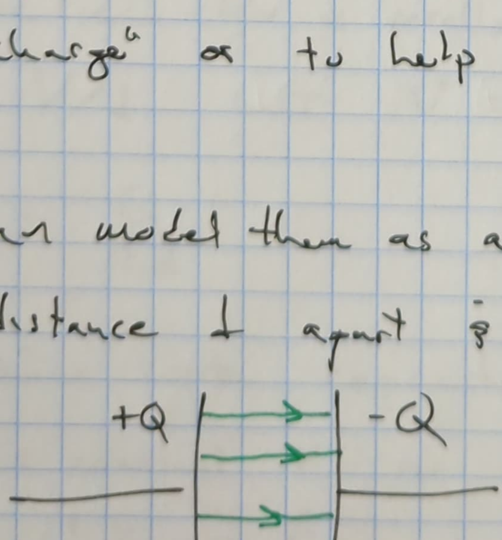
Also, the surface of a conductor is, by definition, an equipotential. ALWAYS!

We tend to model the earth as an equipotential (why?) & typically use this as our reference point; the "ground"
If our HV circuits weren't grounded then touching one could be deadly.

Capacitors

A capacitor is a device we model as "storing charge" or to help "even out" an electric signal.

We can model them as a pair of thin flat plates held a distance d apart & connected to a battery.



The amount of charge on each plate is a function of the potential difference, distance, geometry, & a value called the "dielectric constant" or "permittivity" of the material b/w the plates.

we write all this as

$$Q = CV$$ where C is called the capacitance of the capacitor.

for a parallel plate capacitor

$$C = \varepsilon \frac{A}{d}$$

if there is purely nothing b/w the plates then

$$\varepsilon = \varepsilon_0 = 8.85 \times 10^{-12}\ F/m = 8.85 \times 10^{-12}\ C^2/Nm^2$$

F is a new unit of capacitance call a Farad.

typical values are μF to fF. 1 F is quite large!

The electric field b/w a parallel plate capacitor is

$$E = V/d \quad \text{but} \quad V = Q/C$$

so

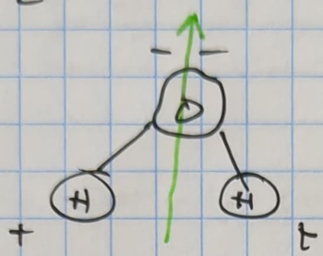
$$E = \frac{Q}{Cd}$$

we sometimes write capacitance in terms of a "dielectric constant" κ as

$$C = \kappa \varepsilon_0 \frac{A}{d}$$

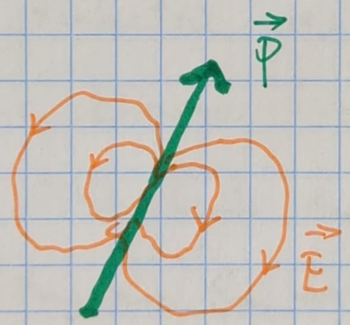
Dielectrics

Dipole if we have a ⊕ & ⊖ separated by a distance d we have what is called an electric dipole.
For example H_2O has a permanent dipole



The green arrow is the dipole

The E-field of a dipole, when viewed far away looks like,



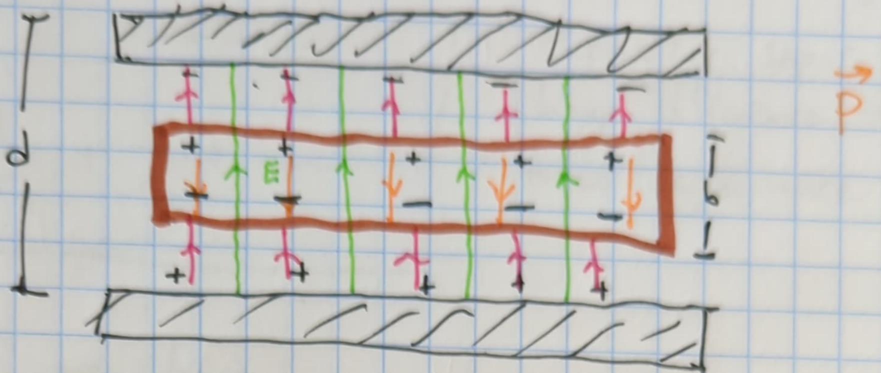
A dielectric is an insulating material that can be polarized by an E-field ~~Water~~

When we insert a dielectric into a parallel plate capacitor the capacitance increases. it can store MORE charge.

or we can say that the voltage is lower for the Same charge.

$$C = \frac{\kappa \varepsilon_0 A}{d} \quad ; \quad Q = CV$$

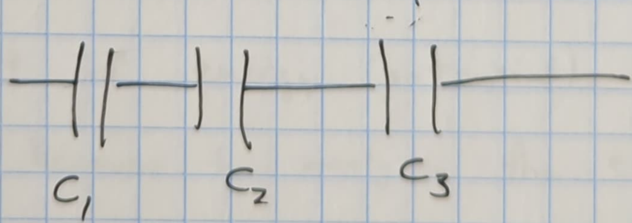
what is going on?



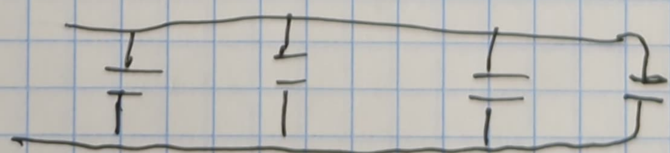
the polarization partially cancels the field. The ~~easi~~ easier it is to polarize a material the larger $\vec{p}$ will be & the lower the $\vec{E}$-field will be. The voltage lowers when the dielectric is inserted. This means we can fit more charge into a capacitor with the same voltage. This leads to the concept of polarization charges. they are the magnitude of the polarization vector.

Capacitor circuits

Series:



parallel:



series capacitors are equivalent to one large capacitor separated by a large difference. thus the total capacitance is reduced.

$$\boxed{\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots}$$

in parallel we have

$$\boxed{C_p = C_1 + C_2 + C_3 + \cdots}$$

the total capacitor is equivalent to one large cap.

<u>Energy.</u>

Capacitors store charge & can store a ton of charge. The first capicitors were called Leyden jars. they were quite literally jars of charge. if you opened the wrong one you might get a HUGE dose of charge & die.

we naturally are lead to the idea that caps have a certain amount of energy (potential). This is a delicate question because to analyze the energy of a cap as we add each little charge to it.

let us see what work the battery does when charging a capacitor to voltage V & charge Q

each little charge we call dq, think of it as an e^-

when we add dq we must do work of

$$dW = V\,dq$$

when we add all of these up we get the total work. we will simply take the average

$$\boxed{W = U = \frac{1}{2} Q V}$$

~~the charge Q V~~ the voltage $V = \frac{Q}{C}$

$$\boxed{U = \frac{1}{2} Q^2/C = \frac{1}{2} Q V = \frac{1}{2} C V^2}$$

This is kind of weird! The battery kicks out QV_b energy but we end up with half of this! where did it go?!

The lost half is heat due to resistance of the circuit. So we simply lower the resistance! but this wont work because the power consumed is $I^2 R$. the heat shoots up as we lower R due to I.

Calculus will eventually clear things up!