An engine that runs on Information

(1) What is information?

Info is a notoriously difficult concept to pin down. In our everyday lives we associate it with "knowledge": but who's knowledge? what knowledge? what can we "know"? what don't we know? the study of knowledge is a field of philosophy called "Epistemology" and it is FASCINATING! we have more meager aims here.

Info has a specific meaning in mathematics. In probability theory every random event is assigned a "chance" called the "probability such-and-such happens".

E.G. if you roll a dice you have a "chance" to roll a "6". If you roll many many times you will find that $\frac{1}{6}$ of the time you will find a "6" face up if the dice is "fair" this is the "frequency" view of probability. It requires you to imagine performing an infinite # of rolls to find the probability of 6 to be $\frac{1}{6}$

If you have N equal options then;

$$p(n) = \frac{1}{N}$$

and $p(n)$ is defined such that

$$\sum_{n}^{N} p(n) = 1$$

i.e

$$p(1) + p(2) + \ldots p(6) = \frac{1}{6} + \frac{1}{6} + \ldots + \frac{1}{6} = 1$$

sometimes $p(n)$ can be some complicated function with unequal weights or it can be continuous rather than discrete! e.g. what is the probability a spinner lands at angle $\alpha = 22°$? there is sort of an infinite # of options so we ask "what is prob ~~the~~ spinner lands within $\pm 1°$ of $22°$?" this is called probability density but we wont need continuity here!

The other view of probability is called "Bayesianism" and probability is ~~as~~ defined as "an amount of belief such & such will happen". This view is surprisingly mathematically well defined via the Cox axioms.

Physics ~~is~~ usually takes a "frequentist" approach. But I, personally, am a Bayesian. They're ultimately equivalent and both suffer various philosophical faults.

Now, if you have a probability distribution $p(n)$ then there exists an important function called the "Shannon Entropy"

$$\boxed{H = -\sum_n p(n) \log p(n)}$$ log base 2

E.G. entropy of a coin-flip is

$$-\left[\tfrac{1}{2}\log\tfrac{1}{2} + \tfrac{1}{2}\log\tfrac{1}{2} \right] = \log 2 = 1$$

we call $\log 2$ a "bit".

like entropy in physics there are several interpretations of shannon entropy. On one hand it represents "the <u>uncertainty</u> <u>before</u> you flip a coin"

On the other hand it represents the "Information you gain <u>AFTER</u> flipping a coin"

In the literature folks use "uncertainty" & "information" interchangeably leading to VAST confusion! Here we will say H is the "info"

If $p(n) = \frac{1}{N}$ then $\boxed{H = -\sum \frac{1}{N} \log \frac{1}{N} = \log N}$

i.e. you get more info from rolling a dice than a coin.

Binary Entropy

suppose you ask a "Yes/No" question

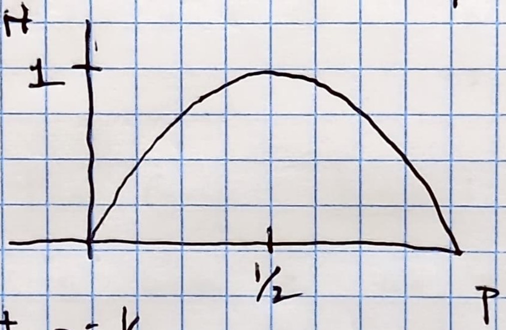
& $p(YES) = p$ then because $\sum p(n) = 1$

$p(NO) = 1 - p$, here p may not be equal to $\frac{1}{2}$

then ~~$H(p) = -\sum p \log p$~~

$$H = -\sum p(n) \log p(n) = -\left[p \log p + (1-p) \log (1-p) \right]$$

a graph of H as a function of p looks like



It has a maximum at $p = 1/2$

& zeros for $p = 0, 1$.

Intuitively, if you ask a question you already <u>know</u> is YES then clearly you don't acquire any information, you have ZERO uncertainty. You get the <u>most</u> info if YES/NO are equally likely.

All this shannon entropy stuff leads into the entire subject of "Information theory", data compression, transmission, languages!, cryptography, coding, & much MUCH more. This is all we need for now.

Thermo

Suppose we have a box of gas with a moveable wall below in a bath of temp T

if we compress from $V_1 \to V_2$ we do work on the gas $\delta W = p\,dV = \frac{NkT}{V} dV$

$$\Rightarrow \quad W = \int_{V_1}^{V_2} \frac{NkT}{V} dV = NkT \log \frac{V_2}{V_1} < 0$$

is the work the Gas does: opposite of our work. Here log is base e we will write ln

the system is iso-thermal & therefore the internal energy of the gas is unchanged: the heat from our work went into the bath

We will choose the "Free Energy" thermo-potential to analyze

$$F = U - TS$$

$$\delta F = \delta U - T\delta S \quad \text{if we change } T \text{ slightly}$$

Here, $\delta U = 0 \Rightarrow \delta F = -T\delta S = -\delta Q$

δF is the heat that went into the bath from our work!

But $\quad -\delta Q = -W = -NkT\ln\frac{V_2}{V_1}$

$$\boxed{-T\delta S = NkT\ln V_1/V_2 \;\&}$$

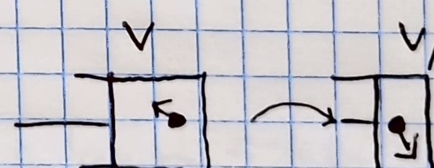
$\Rightarrow \quad \Delta S = \cancel{Nk\log}\; Nk\ln V_2/V_1$

which is Less than zero, But doesn't entropy always increase? Not here because WE input energy to lower the entropy. The gas is confined to a smaller volume & therefore "more ordered" rather than disordered!

★ The Leap of Faith ★

suppose $N=1$; a box with ONE molecule or better a single particle with no structure. Further, let's take $V_2 = V_1/2$; we compress by half; then what?

well $N=1 \quad V_2 = V/2$



plugging in

$$\boxed{\begin{aligned}\Delta S &= \cancel{k\ln}\; -k\log 2\\ \Delta F &= kT\log 2\end{aligned}}$$

This is MASSIVELY DEEP & not obvious but this is hinting at us to change our perspective.

Before you Do work you ask "Is the particle on the left or right side of the box?" By symmetry this is identical to a coin flip It could be in either with prob(left) = prob(right) ie $p = 1/2$, $1-p = 1/2$

Now, <u>we</u> do work & push in the piston & Now WE <u>KNOW</u> it is on the right side BY DOING WORK WE GAIN INFORMATION!

Shannon tells us

$$H = \log 2 = 1 \text{ bit}$$

~~But~~ Boltzmann tells us

$$\Delta S = k \log 2 = k \ln 2$$

but k is just a conversion factor, a fundemental constant that relates energy & temperature J/K sometimes ln 2 is called a "nat" we will use the word "bit" for it as well.

We have thus boldly discovered a "physics definition" of information.

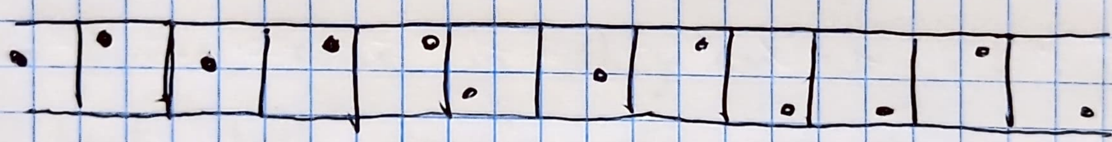
It is the amount of FREE ENERGY needed to compress $V \to V/2$.

Let's call $\frac{V}{2}$ the state "0"

& V as the state "1"

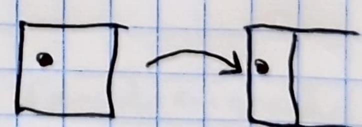
★Building an engine that runs on pure info ★

Let's build the following



It's a long chain of the boxes with moveable pistons.

Importantly if <u>YOU</u> <u>KNOW</u> the object is on the left side of the box than you can compress with ZERO work



theres no resistance, If it's on the right then just move the other wall "for free"

It's only when "we don't know" where it is that we do work. Sure, we might get

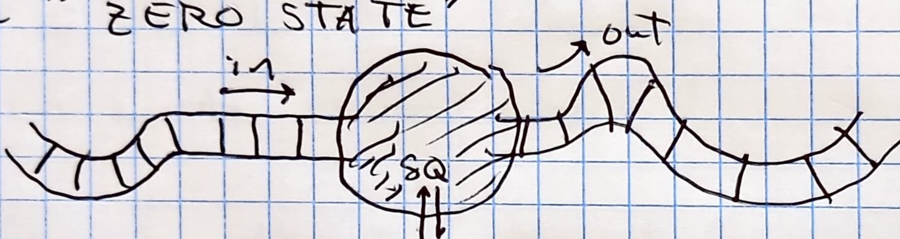
Lucky but for MANY Boxes that luck gets washed out & we must do work on average.

let's build, following Charles Bennet ~1985,

a machine that uses info as ~~fu~~ fuel.

we need a "bath" at temperature T

our machine takes a string of boxes all in the "ZERO STATE"



recall 0 means $V/2$

as the boxes come in they are warmed up by the heat bath causing the gas to expand against the piston causing $\frac{V}{2} \rightarrow V$

the total work is just $kT \ln 2$ per box

for N boxes we get

$$W = NkT \ln 2 \quad J$$

the tape that comes out of fully randomized we have NO MORE INFORMATION about the atoms in the boxes. We HAD that info & used it to do useful work!

It's ~~an~~ a pristine example of the dictum

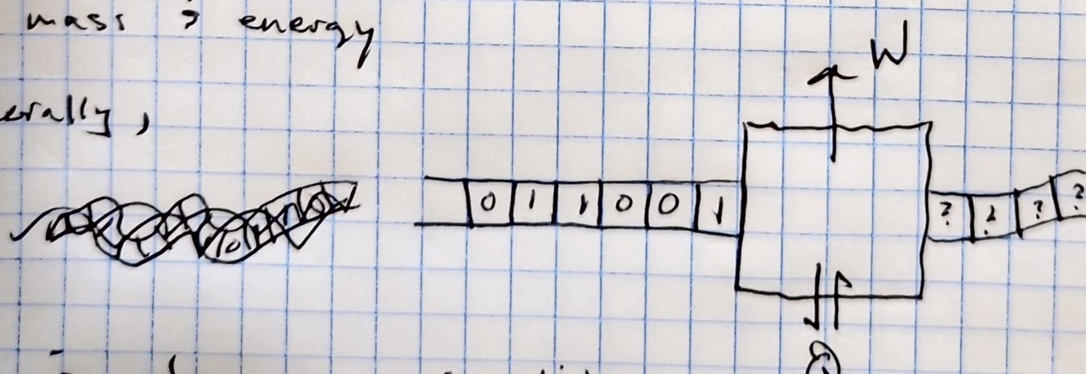
KNOWLEDGE <u>IS</u> POWER!

① if we don't know which side of the box the atom is on the WE do work to <u>gain</u> information (an experiment)

② if we do have information then it can be used to extract work!

This is ultimately general from our box model much like Carnot was independent of his model.
This is a fundamental link between information & energy similar to einstein's link b/w mass & energy

Generally,



There is also a major link between gravity, space-time, & information. there is a limit to how much Info can exist at a point in space!